Inertial Sensors- A New Approach for Low Cost Calibration and Testing

Z. Berman

Berman Consulting and Training Limited (berman.consult@gmail.com)

Moshavat Kinneret 151105

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Abstract

This paper proposes a new approach to calibration of inertial sensors. Instead of requiring stable and precise reference data, we address calibration (and testing) during changing angular rates and temperatures. This approach reduces significantly the equipment cost and calibration time, which is often a critical factor in calibration cost. Moreover, due to the capability to collect dense data, with respect to temperature, it may provide better performances.

The proposed scheme is based on three (two) axis low cost rate table with the capability to know relatively precisely the gimbal position, and with a very rough requirement to control the rate of the reference gimbals system. Similarly, we assume climatic chamber without tough requirement to set a specific, stable temperature. The idea is to run the system along its temperature range with a moderate temperature gradient (0.5-2 °C/min, related to the operational life) and to expose the system to different kinematic cycles (static and dynamic) to allow observability of error model parameters. We do not require rate measurements during dynamic movements, initial and final angular positions provide the necessary reference data. This proposed scheme will be referred to as the “gradient method”.

For model based calibration, where some structure for error function is assumed and the calibration is reduced to parameter estimation, the key is in the systematic design, analysis, and advanced algorithms (Kalman Filter) application. In this paper, two approaches (standard method and the gradient method) are compared and analyzed for a specific example, and the performances advantage of the proposed scheme is shown. Due to the big amount of collected data, the proposed approach can be used even without any strong assumption about the structure of the error function; in this case the key is to define the parameters to be estimated as mean values over carefully selected ranges of temperatures and rates and to perform some smoothing on the resulting estimations. Both methods, their design, analysis and implementation, are based on an error model, essentially reliable and precise instability model. The standard approach based on Alan Variance is not appropriate here; a different approach called Direct Bound is proposed. It is based on finding markov process parameters, such that the underlying markov process bounds the angular (velocity) errors for a wide range of times, calculated by integrating the row data corrected by bias, based on data from a fixed (usually relatively short) interval.

As a result, by combing a reliable error model, a novel scheme for inertial sensor calibration, advanced algorithms and analysis tools, a unified and efficient framework for system calibration, testing and modeling is established.

1. Introduction

When reviewing the literature on inertial sensor calibration it stands out that papers from well established navigation companies are missing; presumably because those companies consider calibration procedure details as their intellectual property. In this context, the attempt of the IEEE standard group in creating a common terminology and framework is very promising. The standard approach (see IEEE Standard Specification Guide [3]) divides error sources into two groups: stochastic and environmental. For stochastic errors, there is a significant effort to create a proper stochastic model, using Alan Variance and related PSD's (Power Spectrum Density). These types of models, such as flick noise named bias instability, are well adapted for time-invariant, stationary systems , work well for room temperature but are not appropriate to deal with environmental (mainly thermal) sensitivities. Our approach is to consider environmental error sources as critically important and divide them further into deterministic errors which can be calibrated and to consider residual errors remaining after calibration as stochastic ones.

The general methods for calibrations were described by in [1] and [2]. The attempt to identify markov process that bound the sensor instability using Alan Variance and frequency domain analysis was proposed in [5], we use the same idea to find a markov process that bounds the sensor instability but our calculation is based on drift corrected integral, which is time domain analysis.

Since a standard, wide accepted error model is not yet available, let's begin by introducing the error model used in this paper, in the sequel the focus will be on gyro, nonetheless, the complete analysis can be easily adapted to accelerometers.

The general gyro error model is described in the following equation:

$$\delta\omega(T,\omega,t) = B(T) + \varepsilon_{d2d} + \varepsilon_{ins}(t) + M(T,\omega)\omega + \varepsilon_{res}(T,\omega,t)\omega + \varepsilon_{RW}(t) \qquad (1)$$

The vector gyro error is the error in measured angular rate $\delta\omega(T,\omega,t)$. It is at least a function of temperature (T), angular rate vector (ω), and time (t). In some models additional sensitivities are presented, for example: sensitivity to magnetic field, linear acceleration, acoustic noise or vibrations. Those sensitivities require specific set-up for

calibration, therefore should be discussed separately. In this paper the focus is on calibration related to the model presented in equation (1).

In equation (1) all terms can be combined into two groups: Ones related to static tests, called drift terms and those related to angular movement, called (extended) scale factor terms, these are separated during dynamic tests. The drift error (the totals gyro error while zero input is applied) is covered by a sum of four terms: $B(T)+\varepsilon_{d2d}+\varepsilon_{ins}(T,t)+\varepsilon_{RW}(t)$, where $B(T)$ is deterministic change of bias with respect to temperature, ε_{d2d} stands for long term instability and day to day change in constant drift (it is assumed to be constant during continuous operation), $\varepsilon_{RW}(t)$ refers to white noise error (in rate!), called random walk (as related to angular error), and $\varepsilon_{ins}(T,t)$, usually named the instability term, covers everything else: drift changes which are not random walk, drift changes due to non- repeatable part of temperature sensitivity, or non modeled thermal effects like sensitivity to thermal gradients (in time and space). The extended scale factor terms (gyro errors related to gyro input only) are covered here by two terms: $M(T,\omega)\omega,\varepsilon_{res}(T,\omega,t)\omega$. $M(T,\omega)$ is deterministic function of scale factor error due to temperature error, it is assumed to be also a function of angular rate vector because this type of error is not necessary linear in angular rate. $\varepsilon_{res}(T,\omega)$ stands for residual extended scale factor error, to cover for example changes in scale factor due to turn on, scale factor stability with respect to time, residual, non- repeatable part of temperature sensitivity, or non modeled thermal effects like sensitivity to thermal gradients (in time and space).

In the context of this paper, the purpose of inertial sensor calibration is to find two deterministic functions: bias $B(T)$ and extended scale factor $M(T,\omega)$. The basic principle is very simple: apply known angular rate ω, calculate the gyro error $\delta\omega(T,\omega,t)$ (by subtracting gyro output from the known reference angular rate), measure the temperature T and separate the deterministic parts $B(T),M(T,\omega)$ from all other components: $\varepsilon_{d2d},\varepsilon_{ins}(t),\varepsilon_{res}(T,\omega,t)\omega,\varepsilon_{RW}(t)$. It turns out that this separation is not trivial at all. The next section is devoted to static calibration, as well as analysis of the separation of bias $B(T)$ from random walk $\varepsilon_{RW}(t)$ and from the instability term $\varepsilon_{ins}(t)$.

2. Static tests and calibration

The system is set at rest; no dynamic movements with respect to the reference gimbal system, nevertheless it measures Earth rates. We need to make the following two assumptions:

1. The Earth rate, in body axes is known and corrected. This means that the angular orientation of the system is known.
2. The influence of earth rate on extended scale factor error is negligible.

After accepting the above assumptions, the error model for the static test is reduced to:

$$\delta\omega(T,\omega,t) = B(T) + \varepsilon_{d2d} + \varepsilon_{ins}(T,t) + \varepsilon_{RW}(t) \tag{2}$$

In the sequel, ε_{d2d} is assumed to be a constant number, which is valid for continuous operation. A standard method to reduce the influence of random walk is to integrate the sum of gyro outputs, which after taking into account earth rate corrections, are gyro errors. The longer the integration time is, the smaller the relative influence of random walk becomes.

The required integration time can be easily calculated from the known standard deviation of random walk and required accuracy of drift measurement ($B(T) + \varepsilon_{d2d} + \varepsilon_{ins}(t)$). Equation (3) describes this accuracy $D_{accuracy}$ as a function of random walk standard deviation σ_{RW} and integration time t_{ing}.

$$D_{accuracy}\left[\frac{^\circ}{h}\right] = \sqrt{\frac{3600}{t_{ing}}} \cdot \sigma_{RW}\left[\frac{^\circ}{\sqrt{h}}\right] \tag{3}$$

For example, let's assume a random walk of $\sigma_{RW} = 0.1\frac{^\circ}{\sqrt{h}}$. The following plot presents the drift accuracy against integration time t_{ing}.

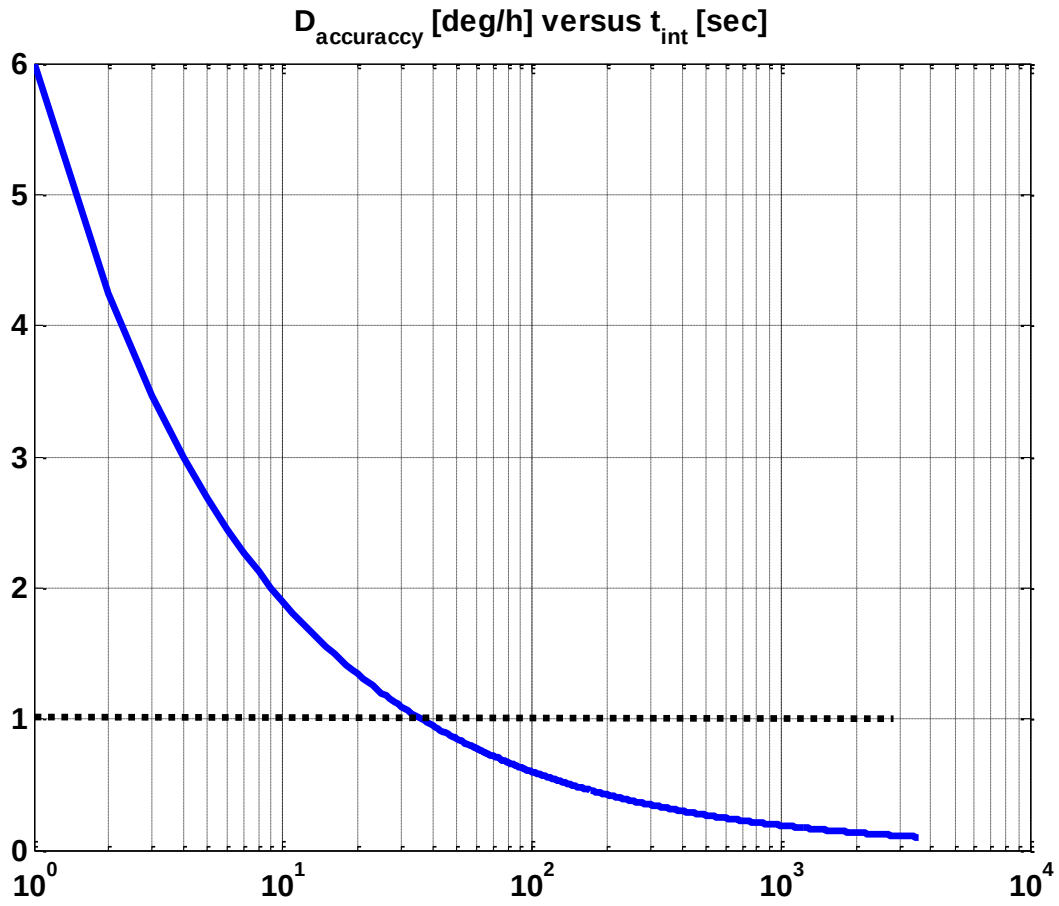

Figure 1: Drift accuracy against integration time

From Figure 1, one can clearly see that in order to achieve 1 $^{\circ}/_{h}$ accuracy, an integration time of 36 seconds is required. This is an important consideration, when calibration process is designed.

Example:

Let's assume that the instability term is described as markov process with standard deviation of 1°/h, and time constant of 120 seconds. That standard deviation of random walk is 01 °/sqrt(h). The temperature range is [-30.70] °C. $B(T)$ is assumed to be a general cubic function:

$$B(T) = a_3\left(\frac{T-20}{50}\right)^3 + a_2\left(\frac{T-20}{50}\right)^2 + a_1\left(\frac{T-20}{50}\right) + a_0 \qquad (4)$$

Two methods will be compared.

Method one: Six stable temperatures will be set: $T_k \in [-30,-10,10,30,50,70]$℃ Good stabilization requires an hour wait at every temperature, followed by half an hour of data collection. Altogether the test lasts for 9 hours.

By integrating (for 1800 seconds) markov process and angular random walk we can calculate the measurement errors. After 0.5 hour of integration, over the given random walk and markov process the angular error (1-sigma) is 0.19°, which is equivalent to a drift of 0.38°/h. This number can be calculated as simple covariance propagation (recommended) or by Monte Carlo simulation. Now, best fitting of 6 points into a cubic structure should be carried out.

Method two; Temperature will be set for -30℃, wait for stabilization for 1 hour and then proceed with applying $1°C\ per\ \min$ gradient, so the test will last 2 hours 40 minutes; Dense measurements will be taken (sum gyro output for every second), set up Kalman filter states that parameters of cubic function are augmented by a model of markov rate and angular error.

The results of the residual errors, due to errors in $B(T)$ parameters, are presented in Figure 2. The results may seem surprising. The standard, conservative method, much more expensive in time, provides significantly less precise calibration results. The main reason is that in the second, novel, method very dense data is collected with respect to temperatures (and time). In general, the measurements contain more noise than the corresponding ones in the first method. Nevertheless, having known the general models for temperature sensitivity and bias instability, we are able to estimate quite precisely the required parameters for temperature sensitivity.

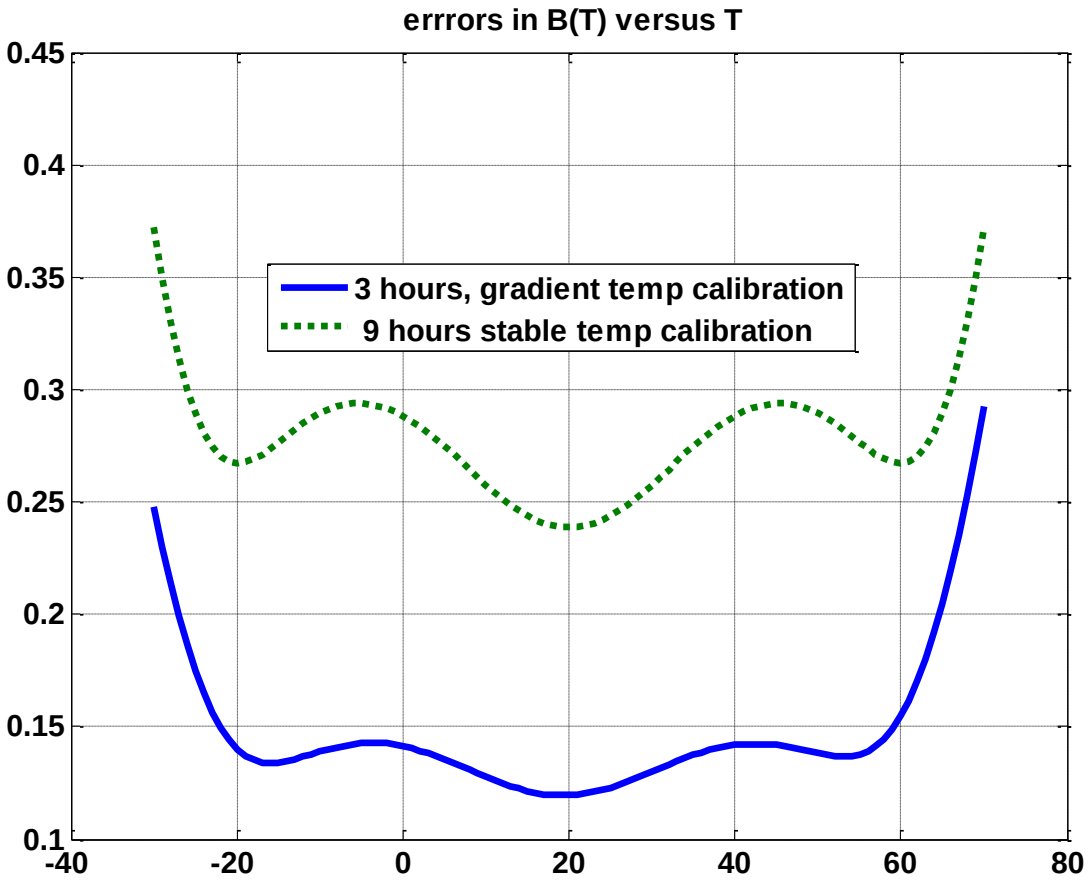

Figure 2: Estimation error comparison - long stable and short gradient calibrations

Besides the performance advantage described above, there are other, much more important advantages of the proposed. However, first, let us admit that the advantage in performance is not necessarily valid in real life. The reason being, is that the assumption of equal instability for both stable and varying temperatures, in most practical situations is not true. Usually when the varying temperatures are applied, the instability becomes much higher. This is perhaps the main rationale to invest so much time awaiting temperature stabilization, hoping to get very good results for gyro instability at constant temperatures. However, during operation life, almost every system is exposed to changes in temperature. Our point of view is that if the system is exposed to temperature changes during its operational life, it should be covered in its calibrations and tests. On the other hand, there is no need to do so for temperature gradients higher than the typical ones.

Figure 2 can be interpreted as follows: if during a temperature gradient of 1°C/min, the instability is 1°/h, the gradient method provided a calibration accuracy of about 0.2°/h, which is satisfactory in the context of 1°/h instability.

The gradient temperature method is able to collect dense data in reference to time and temperature. For the case that the structure of temperature sensitivity function is given- this data can be used to estimate the required parameters. However, in the case that the structure of sensitivity function is unknown, we can use the dense data to build the sensitivity function using very weak assumptions, for example assuming only some smoothness of the sensitivity function. Notably, This task cannot be carried out using the standard method, where only a few points of stable temperature measurements are available.

If we are given the instability error model, we can design and optimize the calibration process. As this is not a clear and straight forward procedure, in the next section we will elaborate on how to create and verify a reliable instability error model.

3. Instability error model – Direct Bound Approach

In the previous section example, the instability term was modeled as markov process with a known standard deviation and time constant. In this section, a new method to measure and verify such a model is proposed. For the sake of simplicity, let us assume that the deterministic temperature sensitivity function $B^E(T)$ was already estimated. The instability will be defined with respect to this estimate (we still use a null input assumption):

$$\delta\omega(T,\omega,t) - B^E(T) = \delta\omega^*(T,t) = \varepsilon_{d2d} + \varepsilon_{ins}(T,t) + \varepsilon_{RW}(t) \qquad (5)$$

The left hand side of this equation is given; the term $\delta\omega^*(T,t)$ will be called residual drift. Our task is to find a mathematical (presumably simple) formula to limit the instability term $\varepsilon_{ins}(T,t)$. Observe that, for a given temperature profile, where $T = T(t)$, we can view instability as a function of time.

There are many propositions to solve this problem. These days, the most popular are based on Alan Variance and relations between Alan Variance and different frequency – domain functions (see [3],[4],[5]). This approach is not applicable here, mainly because our model includes varying temperatures which make the underlying process not stationary. This is due to the fact that for different temperature we may expect different instabilities.

Observe that, as presented in Figure 2, even calibration accuracy is temperature depended.

Our goal is to find a markov process that can provide a reliable bound for instability function, valid for every temperature. From the operational point of view, we do not know in what temperature range the system will operate. Calculating mean values over the whole temperature range is related to the problematic assumption that the statistical distribution of temperature is uniform; this assumption is usually not valid. Moreover, this type of approach may hide some strong (or even singular) instability in the specific narrow range of temperatures.

In general, the instability function may include some short term components, and some long term components, and the question how to find the dominating one is not trivial. Alan Variance may provide a good general impression for a wide range of time scales; but it fails to provide precise description of instability due to thermal changes in a specific range of times of interest. This is exactly the aim of the Direct Bound approach.

Our approach is motivated by Alan Variance but with three important modifications:

- Instead of using statistical expectation operator (mean values), the maximal operator is proposed.
- In the Direct Bound, the estimation time is constant and the integration time is a free parameter whereas, the estimation time in Alan Variance is always equal to integration time.
- The bounding model (markov process) is not included in Alan Variance analysis; In our approach, it is an essential part of the method.

Let us describe the Direct Bound method in details. The Direct Bound approach consists of the following steps:

1. First the drift $D(t_0,\Delta)$ will be estimated using data collected during Δ seconds, starting at t_0.

$$D(t_0,\Delta)=\frac{1}{\Delta}\int_{t_0-\Delta}^{t_0}\delta\omega^*(T(s),s)ds \tag{6}$$

2. Secondly, the actual mean rate of angular error during time t will be calculated by integrating the residual drift corrected by the drift $D(t_0,\Delta)$, taking absolute value and dividing it by the time period t.

$$D_\theta(t,t_0)=\frac{abs\left(\int_{t_0}^{t_0+t}\left[\delta\omega^*(T(\tau),\tau)-D(t_0,\Delta)\right]d\tau\right)}{t}\quad t_1<t<t_f \tag{7}$$

Observe that the time period t is limited above by t_1 which may be in the order of one second, we are not interested here in analyzing some effects related to the fact that random walk is not necessarily absolutely white. On the other hand, we limit the time below by a value t_f which may be in the order of 5-20 minutes. Allowing overly long integration times will cause lost of focus on short term components behavior. Working with too long integration times may cause bring us into a region dominated by drift rate where markov process modeling may fail. If a very wide time scale is required, the recommendation is to divide the analysis into two sub-ranges. As a consequence of this limitation we need to remove from the residual drift the long term drift, calculated for times beyond t_f.

3. Calculate the standard deviation of angular process model, divided by the time period. The angular process model is driven by two components: markov process with two parameters: σ_M - its standard deviation and τ_M its time constant, denoted as $M_P(\sigma_M,\tau_M,\tau)$ and angular random walk with fixed parameter (as known from the system specifications or founded separately), denoted as $A_{rw}(\tau)$.

$$D_L(t,\sigma_M,\tau_M)=\frac{\sigma\left(\int_0^t\left(M_P(\sigma_M,\tau_M,\tau)+A_{rw}(\tau)\right)d\tau\right)}{t} \tag{8}$$

The initial condition for $M_P(\sigma_M,\tau_M,\tau)$ reflects the fact that the drift was estimated for Δ seconds. The simplest way to calculate $D_L(t,\sigma_M,\tau_M)$ is to run simple Kalman Filter simulation, with two states, angle and markov process drift, first Δ seconds to assume alignment (angles measurements) and then to let it run in open loop up to t_f.

4. Select the markov process parameters σ^*_M, τ^*_M as minimal values to ensure that indeed $k_S D_L(t, \sigma^*_M, \tau^*_M)$ is a bound for $D_\theta(t, t_0)$ for all $t \in \left[t_1, t_f\right]$ and all t_0. k_S is a statistical parameter relating between standard deviation and actual maximal values, it can be set between 2 and 3 , depends of the required margin (for example for ageing or other tests).

$$\left(\sigma^*_M, \tau^*_M\right) = \arg\min_{\sigma_M} \left\{\sigma_M, \tau_M : k_S D_L(t, \sigma_M, \tau_M) \geq D_\theta(t, t_0) \quad t \in [t_1, t_f] \quad \forall t, t_0\right\} \quad (9)$$

In general, we cannot assure that there is a unique pair $\left(\sigma^*_M, \tau^*_M\right)$ to satisfy the above equation. For multiple solutions we will pick up the one with a more reasonable time constant (usually the shorter one, but can be decided on case by case approach).

So far we defined the markov process parameters that can provide a reliable bound for instability over the relevant range of times. The valid question is how to calculate these parameters. Complex optimization algorithm enthusiasts may be disappointed; due to the rough accuracy of required number and a smooth behavior of limiting function $D_L(t, \sigma_M, \tau_M)$, a simple two dimensional search is satisfactory here. In the sequel, using an example, a general algorithm, to find those parameters is outlined.

First observe that $D_L(t, \sigma_M, \tau_M)$ is not a function of t_0, so let's define:

$$D_\theta^*(t) = \max_{t_0} \left(D_\theta(t, t_0)\right) \quad (10)$$

In real life situations, the range of expected numbers for σ^*_M, τ^*_M is known. Regarding to the previous example let us assume that time constant is in the range $[20, 600]$ seconds and standard deviation is expected to be in the range of $[0.5, 5]$ deg/h. We start with $\tau_M = 100$ and increase the σ_M from 0.5 deg/h (in steps of 0.1 deg/h) until we bound all $D_\theta^*(t)$. This phase is always successful, because $D_L(t, \sigma_M, \tau_M)$ is increasing function of σ_M for all t. Here we will find a value (minimal because we search by increasing the parameter) $\sigma^{100}{}_M$ that bounds all $D_\theta^*(t)$, namely $k_S D_L(t, \sigma^{100}{}_M, 100) \geq D_\theta^*(t) \ \forall t$. Observe that markov process with standard

deviation of $\sigma^{100}{}_M$ and time constant of 100 seconds is a valid (bounding) instability model. The next step is to change the time constant (with steps of at least 10%), by searching around the previous point, we can improve our model (to get lower standard deviation).

Example – Some properties of the limiting function $D_L(t,\sigma_M,\tau_M)$

We assume nominal values $\sigma_M = 1°/h$, $\tau_M = 120\ s$, $\Delta = 10s$ and $\sigma\left(A_{rw}(\tau)\right) = 0.1°/\sqrt{h}$. The following plots present time functions of the limiting function $D_L(t,\sigma_M,\tau_M)$ with respect to parameters: Δ,τ_M,σ_M .

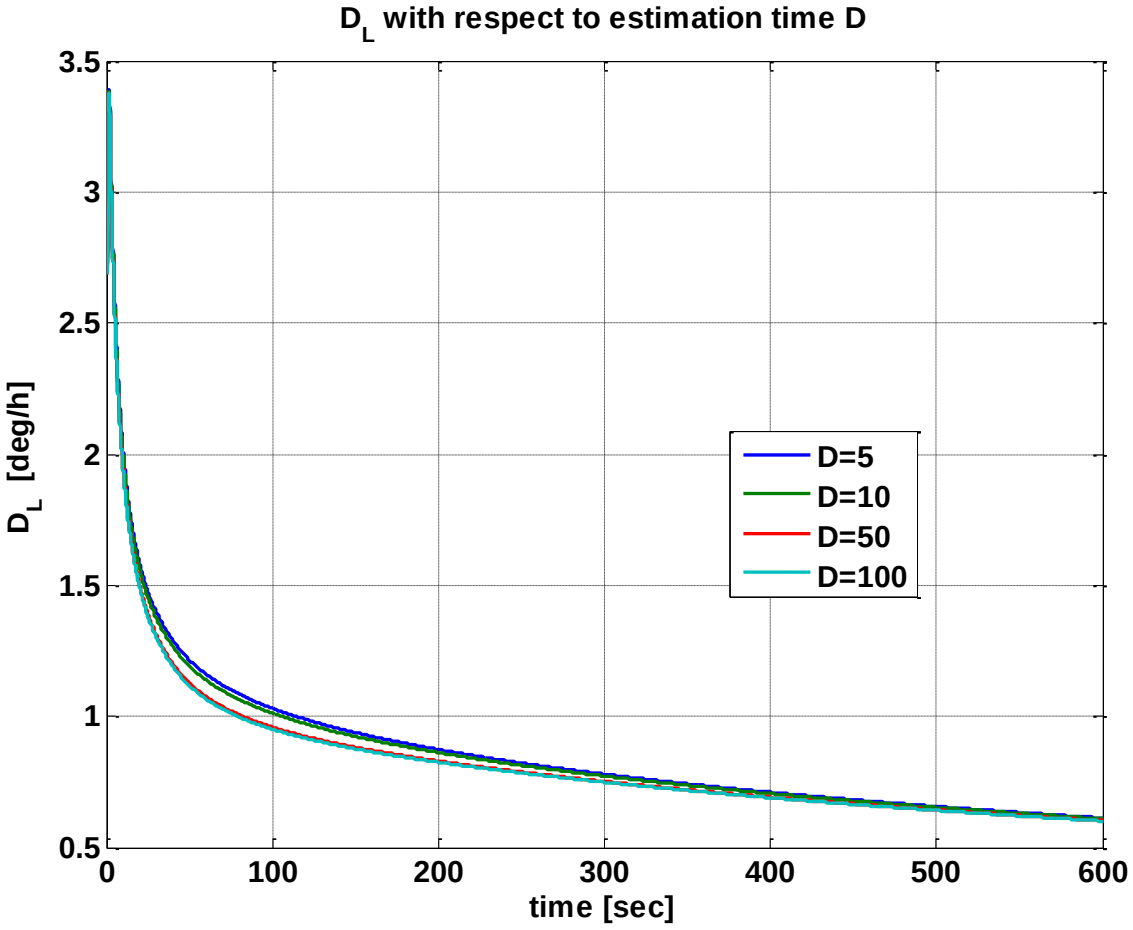

Figure 3 The limiting function $D_L(t,\sigma_M,\tau_M)$ sensitivity to estimation time Δ

One can be surprised as to the extremely low sensitivity of limiting function with respect to estimation time Δ. For longer nominal markov process time constants the sensitivity may be higher.

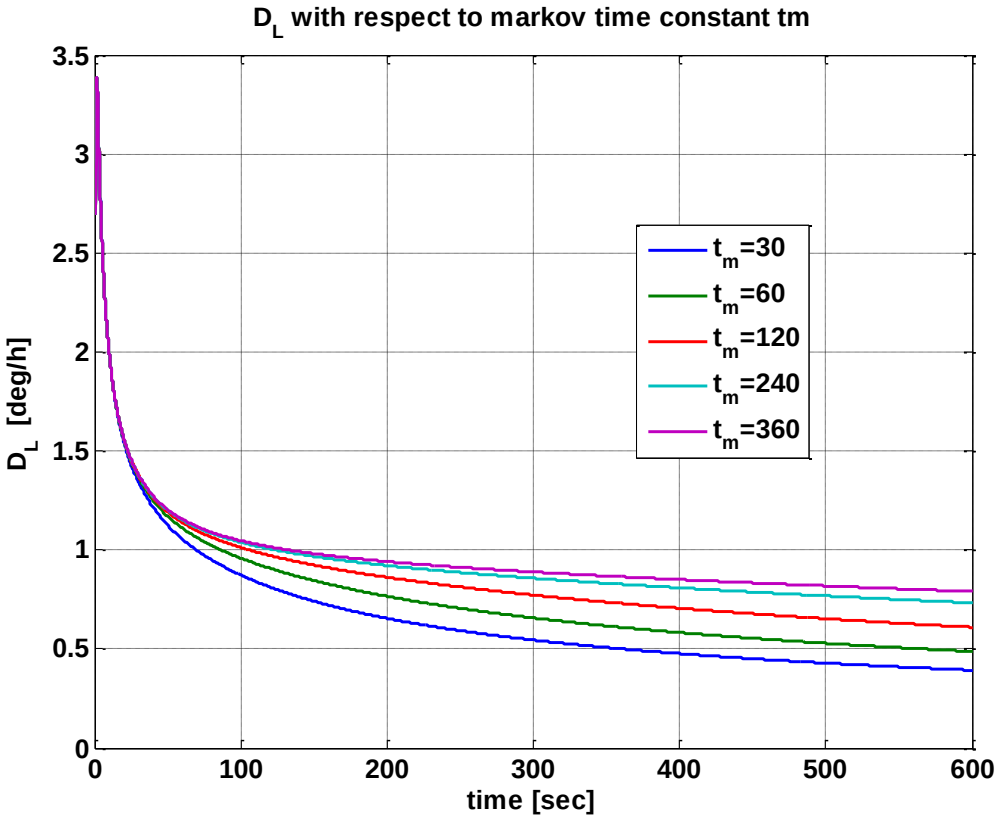

Figure 4 The limiting function $D_L(t,\sigma_M,\tau_M)$ sensitivity to markov time constant τ_M

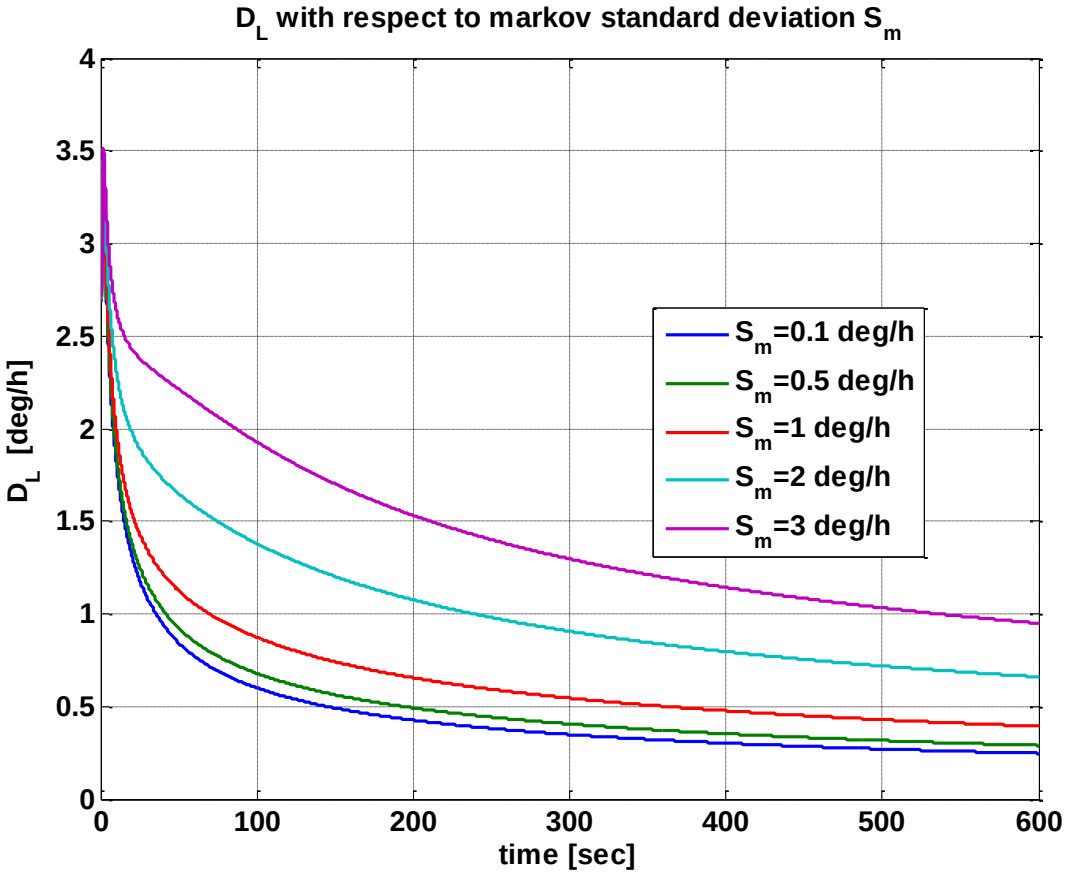

Figure 5 The limiting function $D_L(t,\sigma_M,\tau_M)$ sensitivity to markov standard deviation σ_M

4. Dynamic test and calibration

The proposed scheme is based on three (two) axes low cost rate table with capability to measure precisely the gimbal position, with very rough requirement to control the gimbal rate. In some extremes, the calibration can be carried out using gimbals system without any gimbal motors and servo control instrumentation; this is done by simply moving the system carefully by hands from one position to other[1]. Nevertheless, typically the preferred gimbal system will have rate control with accuracy and stability in the range of 0.1-10%. In any case we do not use rate measurements as reference data, for every movement, the kinematic reference data are simply the initial and final angular positions measured by relatively precise encoders.

The scheme for calibration is simple:

- Apply temperature gradient of $dT\ [°C/\min]$ along the whole operational temperature range.
- For x-axis input:
  - Keep the system at rest for t_{rest} seconds. This period is required to estimate the instability term of the drift.
  - Apply some dynamic profiles to the system $\omega_x = \left\{\omega^k(t)\right\}_{k=1,2,...,K}$ for t_{dyn} seconds. The idea is to get good observability for scale factor errors, even for non-linear setup, but to keep the angular error due to instability limited (see previous section).
- Repeat for y and z-axis.

Accordinly, the design parameters for this type of calibration are:

- temperature gradient dT
- rest time t_{rest}
- dynamic profiles $\omega_x = \left\{\omega^k(t)\right\}_{k=1,2,...,K}$ and their total time t_{dyn}

These are set with respect to:

- sensor error model

[1] Such, relatively limited system was used and tested during my cooperation with Zickel Engineering LTD.

- reference data accuracy
- required accuracy

In the following example, the details of the calibration process design and analysis are presented.

Example:

Consider a scalar scale factor error as 3rd order polynomial function of angular rate (in the range of -300,300 °C/sec), such that every coefficient is itself 3rd order polynomial function of temperature (in the range -30,+70 °C), namely :

$$M(T,\omega)=\left\{b_{33}\left(\frac{T-20}{50}\right)^3+b_{32}\left(\frac{T-20}{50}\right)^2+b_{31}\left(\frac{T-20}{50}\right)+b_{30}\right\}\left(\frac{\omega}{300}\right)^3+$$
$$+\left\{b_{23}\left(\frac{T-20}{50}\right)^3+b_{22}\left(\frac{T-20}{50}\right)^2+b_{21}\left(\frac{T-20}{50}\right)+b_{20}\right\}\left(\frac{\omega}{300}\right)^2+$$
$$+\left\{b_{13}\left(\frac{T-20}{50}\right)^3+b_{12}\left(\frac{T-20}{50}\right)^2+b_{11}\left(\frac{T-20}{50}\right)+b_{10}\right\}\left(\frac{\omega}{300}\right)+$$
$$+\left\{b_{03}\left(\frac{T-20}{50}\right)^3+b_{02}\left(\frac{T-20}{50}\right)^2+b_{01}\left(\frac{T-20}{50}\right)+b_{00}\right\} \qquad (11)$$

The sensor has an instability term modeled as markov process with standard deviation of 1°/h and time constant of 120 seconds, and angular random walk of 0.1 °/sqrt(h).

In this case, the scale factor calibration requires estimating 16 parameters of $M(T,\omega)$: $b_{33},b_{32},b_{31},b_{30},b_{23},b_{22},b_{21},b_{20},b_{13},b_{12},b_{11},b_{10},b_{03},b_{02},b_{01},b_{00}$.

Two methods will be compared:

- The standard one, with stable temperature and angular rate reference with accuracy of 139 ppm. The temperature points are set to: -30,-10, 10, 30, 50, 70 °C, all together 6 points. At every temperature, the following angular rates are applied ±10, ±150, ±300 °/sec, altogether 6 different angular rates. The estimation of $M(T,\omega)$ parameters can be excluded from the instability term, because the integration times are relatively short, which make the instability term negligible. Therefore simple Least Square calculation provides the solution.
- The proposed (gradient) method based on temperature gradient of 1°C/min. The reference angular accuracy is 0.05° (0.85 mrad), for one rotation, it is equivalent to

angular rate accuracy of 139 ppm used in the standard calibration above. One cycle consists of angular profiles based on maximal angular rates of ±10, ±150, ±300 °/sec; every profile provides one complete turn, they are shaped as cosine so they contain all angular rates from zero till the maximal rate. One cycle per axis takes two minutes, with twice 10 seconds of rest and 50 seconds of dynamic movements. After one cycle on a specific axis, there are two cycles (4 minutes) devoted for other axes, and so on. In this case, the best results are obtained using Kalman Filter estimation, which, besides the states related to $M(T,\omega)$ parameters, has two additional states: angular error and instability drift error (markov process).

After completing the estimation process, the covariance of residual error: $\left(M(T,\omega)-M^{E}(T,\omega)\right)$ is calculated. Since the residual error is a function of two parameters: temperature and rate, its visualization is not trivial. In the following plot (Figure 6), the residual errors are presented as functions of angular rate, while every value is taken as maximum over temperature range, once over the whole temperature range (-30, 70 °C), and then over limited temperature range (-20, 60 °C). From figure 6 it is clear that over the reduced temperature range, the gradient method exhibits significantly better performance, gradient method provides about 40ppm average error , while the standard method provides about 100 ppm average error. For the complete temperature range, the mean value of scale factor error for gradient method is 77 ppm, while for the standard method the mean error is 126 ppm. For extreme temperatures (below -20 °C and above 60 °C) and extreme ranges (beyond 270 °/sec), the gradient method is slightly less accurate/ precise than the standard one.

Keep in mind, that calibration time for the standard method may be long as 6 hours, (assuming one hour stabilization for every temperature point) , while for gradient method it will last less than 3 hours (1.5 hour for the test itself and stabilization time for initial temperature).

In addition, the equipment costs are different. To have 0.85 mrad angular accuracy is quite straightforward, while assuring stable and precise reference rate under 150 ppm may call for expensive equipment.

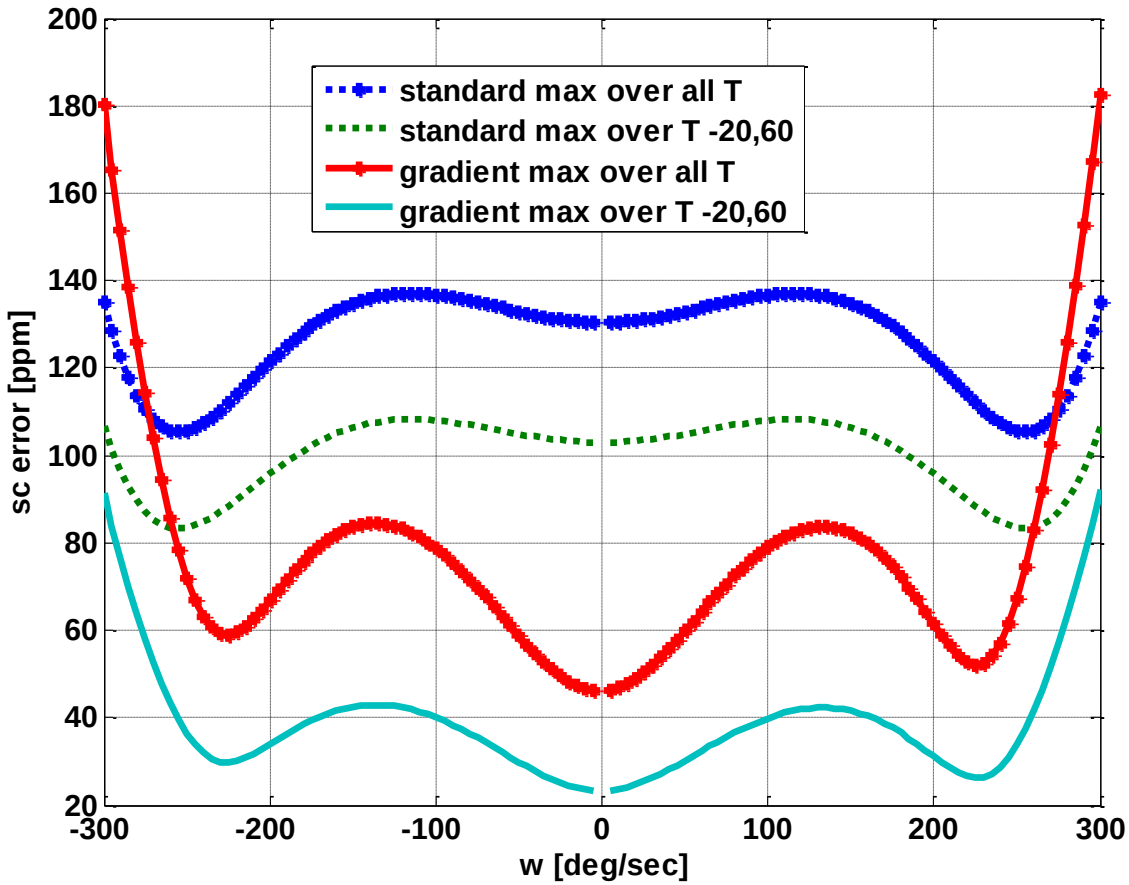

Figure 6 Comparison of residual scale factor errors

So far we have shown that for model based calibration the gradient method outperforms the standard one. Model based calibration is interpreted here as a case where the structure of scale factor error is given and we need only to estimate the underlying parameters.

There are cases for which such structure is not assumed to be known, this situation may occur:

- during initial phase of new sensor evaluation.
- if one wants to develop one calibration process for a wide family of sensors.
- if one works with low quality Mem's.

For such cases the gradient method, for the same calibration time, is able to provide much more dense data (with respect to two dimensional grid of rates and temperatures), however, the estimation technique must to be updated. For those cases, the following recommendations should be considered:

- To deal with changing rates: define the parameter to be estimated as mean value scale factor errors over a range of applied angular rate profile.
- To deal with changing temperature: define the sequence of angular movements such that the related rates are applied close in time (and temperature). For small temperature changes, instead of using linear interpolations, consider to use “closest neighbor approximation”.

5. Conclusions

We presented here a new approach to calibration which achieves the following elements:

- Reduces calibration time by working with varying temperatures.
- Saves equipment costs by working with angular reference data instead of working with reference angular rate.
- Designs the calibration scheme using complete performance analysis based on relevant error model.
- Verifies the instability part of error model using the Direct Bound method.

Using the above method we were able to show that the proposed scheme outperforms significantly the standard calibration scheme.

References

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